PERGAMON

Vision Research 41 (2001) 1683–1684

Vision Research

www.elsevier.com/locate/visres

Letter to the Editor

Reply to letter to editor by M. J. Sankeralli and K. T. Mullen published in Vision Research, 41, 53–55: Lights and neural responses do not depend on choice of color space

Sankeralli and Mullen (2001) argue that the relationship between lights and neural mechanisms depends on which color space is used. In particular, they dispute what they term the 'orthogonality property', according to which modulation of lights orthogonal to the direction of a linear mechanism is invisible to the mechanism. They maintain that the orthogonality property is not valid for any arbitrarily chosen color space. They claim, 'that this orthogonality property was originally applied to a cone contrast space ... and ... is valid only for this space or any orthonormal transformation of this space.'

We disagree with their claim. Lights and neural responses do not depend on the color space used to represent the data. The orthogonality property is not a relationship between lights, but a relationship between lights and visual mechanisms. That a modulation of lights is invisible to a particular detection mechanism does not depend on the color space used to represent this relationship.

A mathematical formulation can help to clarify this relationship. Color spaces used to represent human vision are constructed to give the same coordinates to lights that have the same appearance. A (linear) color mechanism is a (linear) functional defined on the coordinates of this space. The space of linear mechanisms is not the color space, itself, but its dual (Krantz, 1975; Knoblauch, 1995). A linear transformation of the color space in which lights are described induces a related but different transformation of the dual space of chromatic detection mechanisms (Kay, 1998; Lipschutz, 1968). The transformation of the dual space is represented by a matrix that is the transposed inverse of the matrix that represents the transformation of the space of coordinates of lights. The orthogonality between a chromatic stimulus and a detection mechanism is maintained when one takes care to transform both the color space and its dual space properly.

Let us examine this in more detail. Suppose that a particular chromatic modulation is represented by a column vector $v = [v_1, v_2, v_3]^T$ and that a linear detection mechanism is represented by a row vector $m = [m_1, m_2, m_3]$. Suppose further that the chromatic modulation is orthogonal to the mechanism (and so is invisible). This can be expressed by setting the dot product of the two vectors equal to zero.

$$0 = m \cdot v \tag{1}$$

If one now transforms linearly the color space of lights using an invertible 3×3 matrix $\mathbf{A}$, then one must take care to transform the dual space in which detection mechanisms reside by the transpose of the inverse of $\mathbf{A}$. Applying these two transformations to the terms on the right-hand-side of Equation 1 reveals that orthogonality is valid for an arbitrary color space:

$$0 = ((\mathbf{A}^{-1})^T m^T)^T \cdot Av = m(\mathbf{A}^{-1}\mathbf{A})v = m \cdot v \tag{2}$$

Specifically, we do not require that $\mathbf{A}$ be unitary as Sankeralli and Mullen suppose. Again, this is because we are defining a relationship between lights and mechanisms and not between lights.

In our work with sectored noise masking, we were careful to observe the formalisms outlined above in order to obtain results that do not depend on the choice of color space (D'Zmura & Knoblauch, 1998). In particular, we simulated the performance of various detection models numerically in terms of mechanism responses rather than in terms of the color spaces in which data were represented, as detailed in the Appendix (D'Zmura & Knoblauch, 1998). The results of these simulations let us reject the notion that mechanisms along the L & M and S cardinal directions are responsible for chromatic detection as well as the hypothesis that detection mechanisms have nonlinear, narrowband sensitivities. The conclusion that chromatic detection is subserved by multiple linear mechanisms is valid.

PII: S0042-6989(01)00028-1

References

D'Zmura, M., & Knoblauch, K. (1998). Spectral bandwidths for the detection of color. *Vision Research*, *38*, 3117–3128.

Kay, D. C. (1998). Tensor calculations. McGraw-Hill, New York.

Knoblauch, K. (1995). Dual bases in dichromatic color space. In: B. Drum, editor. Color Vision Deficiencies XII, Kluwer, Dordrecht. p. 165–177.

Krantz, D. H. (1975). Color measurement and color theory: I. Representation theorem for Grassmann structures. *Journal of Mathematical Psychology*, *12*, 283–303.

Lipschutz, S. (1968). Linear Algebra. McGraw-Hill, New York

Sankeralli, M., & Mullen, K. (2001). Assumptions concerning orthogonality in threshold-scaled versus cone-contrast colour spaces. *Vision Research*, *41*, 53–55.

Received 16 January 2000

Kenneth Knoblauch
INSERM Unité 371,
Cerveau et Vision, 18 avenue du Doyen Lépine,
69675 Bron Cedex, France

Institut de l'Ingénierie de la Vision,
Université Jean Monnet,
Site GIAT-Industrie,
3 rue Javelin-Pagnon,
B.P. 505,
42007 Saint-Étienne Cedex 01,
France
E-mail: knoblauch@vision.univ-st-etienne.fr

Michael D'Zmura
Department of Cognitive Sciences,
University of California,
Irvine,
CA 92697,
USA